The phase structure of supersymmetric Sp($2N_c$) gauge theories with an adjoint

JHEP11(2009)049

(http://iopscience.iop.org/1126-6708/2009/11/049)

Published by IOP Publishing for SISSA

Received: *October 7, 2009*
Accepted: *October 27, 2009*
Published: *November 10, 2009*

The phase structure of supersymmetric $\mathrm{Sp}(2N_c)$ gauge theories with an adjoint

David Poland

Jefferson Physical Laboratory, Harvard University,
17 Oxford Street, Cambridge, Massachusetts 02138, U.S.A.

E-mail: dpoland@gmail.com

Abstract: We study the phase structure of $\mathcal{N} = 1$ supersymmetric $\mathrm{Sp}(2N_c)$ gauge theories with $2N_f$ fundamentals, an adjoint, and vanishing superpotential. Using a-maximization, we derive analytic expressions for the values of N_f below which the first several gauge-invariant operators in the chiral ring violate the unitarity bound and become free fields. In doing so we are able to explicitly check previous conjectures about the behavior of this theory made by Luty, Schmaltz, and Terning. We then compare this to an analysis of the first two 'deconfined' dual descriptions based on the gauge groups $\mathrm{Sp}(2N_f+2)\times\mathrm{SO}(2N_c+5)$ and $\mathrm{Sp}(2N_f+2)\times\mathrm{SO}(4N_f+4)\times\mathrm{Sp}(2N_c+2)$, finding precise agreement. In particular, we find no evidence for non-obvious accidental symmetries or the appearance of a mixed phase in which one of the dual gauge groups becomes free.

Keywords: Supersymmetry and Duality, Supersymmetric gauge theory, Duality in Gauge Field Theories

ArXiv ePrint: 0908.2131

doi:10.1088/1126-6708/2009/11/049

JHEP11(2009)049

Contents

1 Introduction

It is of great interest to understand the low-energy behavior of asymptotically free gauge theories. Analyzing such theories in the strong-coupling regime is generally quite difficult, but can become tractable with a sufficient amount of symmetry. Strongly-coupled gauge theories with $\mathcal{N} = 1$ supersymmetry are particularly interesting in that they are both amenable to analysis and potentially relevant for phenomenology. Possible applications include dynamical supersymmetry breaking [1–3], conformal sequestering [4, 5], dynamical solutions to the $\mu/B\mu$-problem [6–8], dynamical explanations of the flavor hierarchies [9–11], dynamical solutions to the doublet-triplet splitting problem [12–14], and so on.

When analyzing a supersymmetric gauge theory, a first-order question is to determine what kind of phase the theory flows to at low energies. Possibilities include that the theory is infrared (IR) free, the theory flows to an interacting conformal (non-Abelian Coulomb) phase, the theory has a 'dual' description that is IR free, the theory confines, the theory dynamically generates a superpotential that breaks the gauge group, or that the theory enters a pure Abelian Coulomb phase (see [15, 16] for reviews). In addition, it is possible that the theory enters a 'mixed phase' consisting of decoupled sectors that are in some combination of the above phases. This can occur, e.g., when the theory is in an interacting conformal regime but some set of operators have become free fields and decoupled from the CFT. More exotically, the theory could have a dual description containing a product gauge group in which one gauge group is interacting and one gauge group is IR free. This was argued to occur, e.g., in SU(N_c) gauge theories with an anti-symmetric tensor [17, 18].

An important tool for studying theories in a conformal regime, a-maximization, was introduced by Intriligator and Wecht in [19], and further developed in [20–22]. The idea is that the correct superconformal $U(1)_R$ symmetry can be determined by maximizing

$$a(R_t) = \frac{3}{32}\left[3\,TrR_t^3 - TrR_t\right] \tag{1.1}$$

over all possible trial R-symmetries $R_t = R_0 + \sum_I s_I F_I$, where R_0 is any initial R-symmetry and F_I are the IR flavor symmetries. This is an extremely powerful technique provided that one understands the IR flavor symmetries. Unfortunately, one cannot always identify the IR flavor symmetries as a subset of the UV flavor symmetries – accidental symmetries can arise (see, e.g., [23] for a number of interesting examples).

How can one gain evidence for accidental symmetries? One way is to check whether there are any gauge-invariant operators in the chiral ring of the theory that violate the unitarity bound, given by $R_{\mathcal{O}} \geq 2/3$ for scalar operators [24]. If this bound appears to be violated, then one plausible interpretation is that $\mathcal{O}$ is becoming a free field with $R_{\mathcal{O}} = 2/3$. In this case there is an accidental symmetry associated with rotations of $\mathcal{O}$, and one must include this symmetry when maximizing $a(R_t)$. In practice, this requires one to instead maximize [20, 25]

$$\widetilde{a}(R_t) = a(R_t) + \frac{dim(\mathcal{O})}{96} (2 - 3R_{\mathcal{O}})^2 (5 - 3R_{\mathcal{O}}) \,. \tag{1.2}$$

However, not all accidental symmetries manifest themselves through apparent violations of unitarity. This happens for example in SU(N_c) gauge theories with N_f flavors of vector-like quarks $\{\bar{Q}, Q\}$ in the range $N_c + 1 < N_f < 3/2N_c$ [26]. Since the anomaly-free U$(1)_R$ symmetry is $R_{\bar{Q},Q} = 1 - N_c/N_f$, the mesons $\bar{Q}Q$ appear to violate the unitarity bound in this range and presumably become free fields. However, this is only part of the story, as it is also believed that the entire dual $SU(N_f - N_c)$ gauge group and the corresponding dual quarks are also becoming free fields, yielding many more accidental symmetries. This is not obvious in the original 'electric' description of the theory, but becomes apparent when the dual 'magnetic' description is analyzed. When studying similar theories, it is clearly of great interest to have dual descriptions available that can be studied, as they may contain evidence for the emergence of non-obvious accidental IR symmetries.

In the present work we will use a-maximization to study $\mathcal{N} = 1$ supersymmetric Sp($2N_c$) gauge theory with $2N_f$ fundamentals Q_i and an adjoint A. While the theory with superpotential $W = A^{2(k+1)}$ is fairly well understood [27], the theory with vanishing superpotential has not been as easy to analyze. An attempt to study this theory using 'deconfinement' [28] was made in [29], where it was proposed that the theory has a sequence of dual descriptions, the first of which is based on an Sp($2N_f + 2$) × SO($2N_c + 5$) gauge theory. However, because the U$(1)_R$ symmetry of the theory was unknown it was not possible to determine which operators, if any, gave apparent violations of the unitarity bound as one varies N_f and N_c. Furthermore, it was not possible to explicitly check the dual descriptions for evidence of additional accidental symmetries, as would occur if either of the dual gauge groups were becoming free. As we will see, such an analysis is now possible using a-maximization, and we will here present an attempt to map out the phase structure of the theory. Similar studies of other theories have appeared in [18, 20, 30–33].

This paper is organized as follows. In section 2 we use a-maximization to study Sp($2N_c$) gauge theory with an adjoint. In sections 3 and 4 we perform a similar analysis of the first two dual descriptions of the theory, comparing the results and looking for evidence of accidental IR symmetries. We give concluding remarks in section 5.

JHEP11(2009)049

	SU($2N_c$)	SU($2N_f$)	U(1)$_X$	U(1)$'_R$
Q_i	□	□	$\frac{N_c+1}{N_f}$	1
A	□□	**1**	-1	0

Table 1. Field content of the theory.

2 Sp($2N_c$) gauge theory with an adjoint

We are interested in studying $\mathcal{N} = 1$ supersymmetric Sp($2N_c$) gauge theory[1] with $2N_f$ fundamentals Q_i and an adjoint A. The field content and anomaly-free symmetries are given in table 1. In particular, we are here interested in the theory with vanishing superpotential. It is believed that this theory is in an interacting conformal regime for all $0 < N_f < 2(N_c + 1)$.

The chiral ring of this theory contains the gauge-invariant operators

$$\begin{aligned} T_k &\equiv TrA^{2k}, \quad k = 1, 2, \ldots \\ M_k &\equiv QA^kQ, \quad k = 0, 1, \ldots \end{aligned} \tag{2.1}$$

As mentioned in the introduction, this theory was previously studied using 'deconfinement' in [29], where it was conjectured that the operators M_k sequentially become free fields as N_f is decreased from the asymptotic freedom limit of $N_f = 2(N_c + 1)$ while the T_k remain interacting. It was also noted in [31] that the large $N_c, N_f \gg 1$ limit of this theory will yield the same R-charges as SU(N_c) gauge theory with N_f flavors and an adjoint, which was studied in [20]. Here we will attempt to map out the phase space allowing for smaller N_f and N_c, comparing our results to the conjectures of [29]. In particular, we will find that both the operators T_i and M_i sequentially become free fields as N_f is decreased, with the precise order depending on the value of N_c. This realizes the behavior that was described as scenario C in [29], and thus we will prove that the conjectured behavior (scenario A) is incorrect.

In order to determine the U(1)$_R$ symmetry of the theory, we should maximize $a(R_t)$ subject to the constraint that the mixed $Tr[\text{U}(1)_R\text{Sp}(2N_c)^2]$ anomalies vanish. Recall that an adjoint of Sp($2N_c$) is a two-index symmetric tensor with index $(2N_c + 2)$ and dimension $N_c(2N_c + 1)$. Anomaly cancellation then requires

$$0 = (2N_c + 2) + 2N_f(R_Q - 1) + (2N_c + 2)(R_A - 1), \tag{2.2}$$

or equivalently

$$R_Q = 1 - \left(\frac{N_c + 1}{N_f}\right) R_A. \tag{2.3}$$

[1]Our conventions are such that Sp(2) ∼ SU(2).

In order to determine the R-symmetry, we should then maximize

$$a(R_A) = \frac{3}{32}\Bigg[2N_c(2N_c+1) + 4N_fN_c\left(3\left(-\frac{N_c+1}{N_f}R_A\right)^3 - \left(-\frac{N_c+1}{N_f}R_A\right)\right) + N_c(2N_c+1)\left(3(R_A-1)^3 - (R_A-1)\right)\Bigg]. \tag{2.4}$$

The correct solution to $da/dR_A = 0$ is then given by

$$R_A = \frac{-3(1+2N_c)N_f^2 + \sqrt{16(1+N_c)^3(3+5N_c)N_f^2 - (3+4N_c(2+N_c))N_f^4}}{12(1+N_c)^3 - 3(1+2N_c)N_f^2}, \tag{2.5}$$

where the positive root of the quadratic equation is picked out by requiring that this be a maximum.

Now we can ask the question of which gauge-invariant operator is the first to violate the unitarity bound as we lower N_f from $2(N_c+1)$. It is straightforward to solve the condition $R_{T_1} \leq 2/3$ for N_f. This gives the condition that the operator T_1 is at or below the unitarity bound when

$$N_f \leq 2(N_c+1)\sqrt{\frac{1+N_c}{7+10N_c}}. \tag{2.6}$$

On the other hand, we should check that M_0 does not hit the unitarity bound first. The condition $R_{M_0} \leq 2/3$ is equivalent to

$$N_f \leq 2(N_c+1)\left(\frac{3+6N_c - \sqrt{13+40N_c+28N_c^2}}{4+8N_c}\right), \tag{2.7}$$

which is less than $2(N_c+1)\sqrt{\frac{1+N_c}{7+10N_c}}$ for positive N_c. Thus, $R_{M_0} > 2/3$ within the entire region

$$2(N_c+1)\sqrt{\frac{1+N_c}{7+10N_c}} < N_f < 2(N_c+1). \tag{2.8}$$

When N_f is below this threshold we assume that T_1 becomes a free field, and we should modify the a-maximization procedure according to the prescription given in [20]. Thus, we should now maximize the function

$$a_2(R_A) = a(R_A) + \frac{1}{96}(2-6R_A)^2(5-6R_A). \tag{2.9}$$

Again solving $da_2/dR_A = 0$ yields the the solution

$$R_A = \left(12N_c(1+N_c)^3 - 3(-8+N_c+2N_c^2)N_f^2\right)^{-1}\Bigg[-3(-4+N_c+2N_c^2)N_f^2 + \sqrt{16(1+N_c)^3(-4+N_c(3+5N_c))N_f^2 + (16+N_c(40+N_c(45-4N_c(2+N_c))))N_f^4}\Bigg], \tag{2.10}$$

JHEP11(2009)049

where the positive root is again picked out by requiring that this be a maximum.

Now that we know the R-symmetry in this region, we can determine which operator is next to hit the unitarity bound. It is straightforward to show that the condition that $R_{T_2} \leq 2/3$ is equivalent to

$$N_f \leq 2(N_c+1)\sqrt{\frac{N_c(1+N_c)}{-24+N_c(37+58N_c)}}. \tag{2.11}$$

This should be compared to the condition for $R_{M_0} \leq 2/3$, which occurs when

$$N_f \leq 2(N_c+1)\left(\frac{-12+3N_c+6N_c^2-\sqrt{16+N_c(-88+N_c(-67+4N_c(10+7N_c)))}}{4(-8+N_c+2N_c^2)}\right) \tag{2.12}$$

It is then easily verified that $R_{M_0} > 2/3$ in the entire region

$$2(N_c+1)\sqrt{\frac{N_c(1+N_c)}{-24+N_c(37+58N_c)}} < N_f \leq 2(N_c+1)\sqrt{\frac{1+N_c}{7+10N_c}}. \tag{2.13}$$

Below this threshold we can repeat the procedure, treating T_2 as a free field, and maximize

$$a_3(R_A) = a_2(R_A) + \frac{1}{96}(2-12R_A)^2(5-12R_A). \tag{2.14}$$

Solving $da_3/dR_A = 0$ then yields the maximum

$$\begin{aligned} R_A = {} & \left(12N_c(1+N_c)^3 - 3(-72+N_c+2N_c^2)N_f^2\right)^{-1}\Big[-3(-20+N_c+2N_c^2)N_f^2 \\ & +\sqrt{16(1+N_c)^3(-12+N_c(3+5N_c))N_f^2+(144+N_c(522+N_c(813-4N_c(2+N_c))))N_f^4}\Big]. \end{aligned} \tag{2.15}$$

Now we find that $R_{T_3} \leq 2/3$ when

$$N_f \leq 2(N_c+1)\sqrt{\frac{N_c(1+N_c)}{-144+N_c(91+146N_c)}}, \tag{2.16}$$

and $R_{M_0} \leq 2/3$ when

$$N_f \leq 2(N_c+1)\left(\frac{-60+3N_c+6N_c^2-\sqrt{144+N_c(-600+N_c(-323+4N_c(10+7N_c)))}}{4(-72+N_c+2N_c^2)}\right). \tag{2.17}$$

The structure this time is somewhat more complicated. If $N_c \leq 28$ then T_3 gives the stronger bound, and if $N_c > 28$ then M_0 gives the stronger bound. However, over a wide range of N_c the difference between these functions is $\lesssim 1$, and M_0 and T_3 hit the unitarity bound at approximately the same values of N_f. In any case, since we are here primarily interested in mapping out the phase structure for smaller values of N_c and N_f, we will first consider the case that T_3 decouples at the larger value of N_f.

JHEP11(2009)049

Treating T_3 as a free field below the threshold in eq. (2.16), we should now maximize

$$a_4(R_A) = a_3(R_A) + \frac{1}{96}(2 - 18R_A)^2(5 - 18R_A), \tag{2.18}$$

which gives

$$\begin{aligned} R_A = &\left(12N_c(1+N_c)^3 - 3(-288+N_c+2N_c^2)N_f^2\right)^{-1}\Big[-3(-56+N_c+2N_c^2)N_f^2 \\ &+\sqrt{16(1+N_c)^3(-24+N_c(3+5N_c))N_f^2+(576+N_c(2544+N_c(3933-4N_c(2+N_c))))N_f^4}\Big]. \end{aligned} \tag{2.19}$$

We can then determine that $R_{T_4} \leq 2/3$ when

$$N_f \leq 2(N_c+1)\sqrt{\frac{N_c(1+N_c)}{-480+N_c(169+274N_c)}} \tag{2.20}$$

and $R_{M_0} \leq 2/3$ when

$$N_f \leq 2(N_c+1)\left(\frac{-168+3N_c+6N_c^2-\sqrt{576+N_c(-2064+N_c(-659+4N_c(10+7N_c)))}}{4(-288+N_c+2N_c^2)}\right). \tag{2.21}$$

Comparing these functions, we find that M_0 decouples at the larger value of N_f for all $8 < N_c \leq 28$. On the other hand, if $N_c \leq 8$, then the bound is only potentially applicable for $N_f = 1$. Thus, M_0 is the next operator to decouple except in the special case of $N_f = 1$, and this decoupling occurs when N_f is below the threshold given in eq. (2.21).

As an immediate check on the results obtained so far, we can expand eqs. (2.17) and (2.21) in the limit of large N_c. Since the effect of decoupling T_i can be neglected in this limit, they should agree up to terms of $O(1/N_c)$. In addition, as was noted in [31], the resulting bound on N_f below which M_0 becomes free should reproduce the results of [20]. We find these checks to be successful. In the large N_c limit we obtain that M_0 becomes a free field when

$$N_f \leq \left(\frac{3-\sqrt{7}}{2}\right)N_c + \left(\frac{3}{2} - \frac{17}{4\sqrt{7}}\right) + O\left(1/N_c\right), \tag{2.22}$$

which does indeed agree with [20] at leading order.

Of course, one can continue this procedure indefinitely. On the other hand, if one is primarily interested in smaller values of N_c and N_f, the region of interest is quickly filled in. In figure 1 we show an approximate phase space diagram for the theory including the next several decoupling thresholds (though we will suppress the analytic expressions). Since the case of $N_f = 1$ is somewhat more complicated, we give its structure separately in table 2.

3 First deconfined dual description

One potential danger with the analysis presented in the previous section is that it may be overlooking non-obvious accidental symmetries that may emerge due to the strong

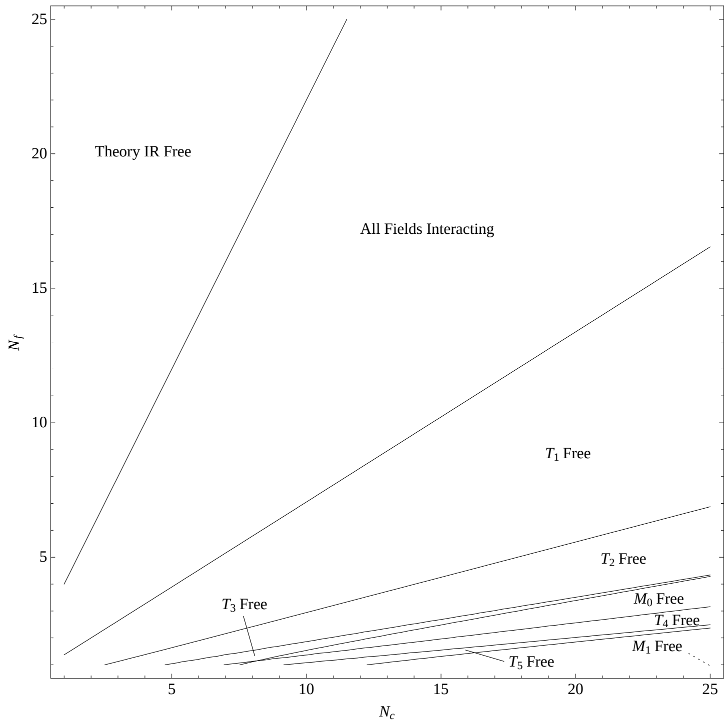

Figure 1. Phase space diagram for $\mathrm{Sp}(2N_c)$ with $2N_f$ fundamentals and an adjoint with vanishing superpotential.

N_c	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
$\mathcal{O}$		T_1	T_2		T_3		T_4	M_0		T_5		T_6	M_1	T_7		T_8		M_2	T_9

Table 2. The values of N_c in the $N_f = 1$ case for which the operators T_i and M_i first become free fields.

dynamics. This happens, e.g., in $\mathrm{SU}(N_c)$ gauge theories with N_f flavors in the range $N_c + 1 < N_f < 3/2N_c$. In that situation, there are accidental symmetries associated with the dual quarks and $\mathrm{SU}(N_f - N_c)$ gauge bosons becoming free fields in the IR. This is manifest when the theory is studied in the dual magnetic description, but completely non-obvious in the electric description. Thus, it is great interest to study dual descriptions of the present scenario in order to look for evidence for a similar scenario.

Fortunately, this theory in fact has a sequence of dual descriptions that can be studied [29]. The dualities are obtained via 'deconfinement', and involve promoting the adjoint A to a composite state in a strongly-coupled $\mathrm{SO}(N_c')$ theory. The first dual description is then obtained by taking $N_c' = 2N_c + 5$ and dualizing the original $\mathrm{Sp}(2N_c)$ gauge group using the known duality for $\mathrm{Sp}(2N_c)$ gauge groups with only fundamentals [34]. The end result of this procedure is given in table 3. In addition, the theory has a superpotential

$$W = M_0 \widetilde{Q}\widetilde{Q} + A_1 \widetilde{x}_1 \widetilde{x}_1 + m_1 \widetilde{Q} \widetilde{x}_1 + m_2 \widetilde{Q} \widetilde{p}_2 + (\widetilde{x}_1 \widetilde{p}_2)(\widetilde{x}_1 \widetilde{p}_2) p_3, \tag{3.1}$$

and the mapping between the gauge-invariant operators in the original and dual description

	$\mathrm{Sp}(2N_f+2)$	$\mathrm{SO}(2N_c+5)$	$\mathrm{SU}(2N_f)$	$\mathrm{U}(1)_X$	$\mathrm{U}(1)'_R$
$\widetilde{Q}$	$\square$	$\mathbf{1}$	$\overline{\square}$	$-\frac{N_c+1}{N_f}$	0
M_0	$\mathbf{1}$	$\mathbf{1}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$2\frac{N_c+1}{N_f}$	2
$\widetilde{x}_1$	$\square$	$\square$	$\mathbf{1}$	$\frac{1}{2}$	1
A_1	$\mathbf{1}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{1}$	-1	0
m_1	$\mathbf{1}$	$\square$	$\square$	$\frac{N_c+1}{N_f}-\frac{1}{2}$	1
m_2	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\frac{N_c+1}{N_f}+\frac{1}{2}-N_c$	5
$\widetilde{p}_2$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{1}{2}+N_c$	-3
p_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$-2N_c$	6

Table 3. Field content of the dual theory.

is given by

$$\begin{aligned} TrA^{2k} &\to TrA_1^{2k} \\ QQ &\to M_0 \\ QA^kQ &\to m_1A_1^{k-1}m_1, \quad k \geq 1. \end{aligned} \tag{3.2}$$

Now, if one just considers the one-loop beta function it naïvely seems that the $\mathrm{SO}(2N_c+5)$ gauge group is IR free for $N_f \geq N_c+1$. However, this is misleading because, e.g., the strong $\mathrm{Sp}(2N_f+2)$ gauge group gives a large anomalous dimension to the bi-fundamental $\widetilde{x}_1$, which in turn gives an $O(1)$ correction to the full $\mathrm{SO}(2N_c+5)$ beta function. This is an example of a larger class of RG flows in product group theories (see [31, 35] for many examples) in which an otherwise IR free coupling can be driven to be interacting because of the other gauge group.

Thus, we will proceed by assuming that the full theory is interacting, again using a-maximization to decide if any fields become free. In particular, we should require that the $\mathrm{U}(1)_R$ symmetry is anomaly free with respect to both gauge groups, i.e. that both the $Tr[\mathrm{U}(1)_R\mathrm{Sp}(2N_f+2)^2]$ and $Tr[\mathrm{U}(1)_R\mathrm{SO}(2N_c+5)^2]$ anomalies vanish. Furthermore, we will start by assuming that the full superpotential in eq. (3.1) is marginal. Notice that anomaly cancellation and the superpotential together give 7 constraints on 8 unknown $\mathrm{U}(1)_R$ charges, causing $a^{\mathrm{dual}}(R_i)$ to again be a function of a single variable as in the previous section.

More concretely, the constraints are

$$\begin{aligned} 0 &= (2N_f+4) + 2N_f(R_{\widetilde{Q}}-1) + (2N_c+5)(R_{\widetilde{x}_1}-1) + (R_{\widetilde{p}_2}-1) \\ 0 &= (2N_c+3) + (2N_f+2)(R_{\widetilde{x}_1}-1) + (2N_f)(R_{m_1}-1) + (2N_c+3)(R_{A_1}-1) \\ 2 &= R_{M_0} + 2R_{\widetilde{Q}} \\ 2 &= R_{A_1} + 2R_{\widetilde{x}_1} \\ 2 &= R_{m_1} + R_{\widetilde{Q}} + R_{\widetilde{x}_1} \\ 2 &= R_{m_2} + R_{\widetilde{Q}} + R_{\widetilde{p}_2} \\ 2 &= 2R_{\widetilde{x}_1} + 2R_{\widetilde{p}_2} + R_{p_3}. \end{aligned} \tag{3.3}$$

These linear equations can be solved to express all of the R-charges in terms of R_{A_1}, yielding

$$\begin{aligned}
R_{\widetilde{Q}} &= \frac{1+N_c}{N_f} R_{A_1} \\
R_{M_0} &= 2 - 2\frac{1+N_c}{N_f} R_{A_1} \\
R_{\widetilde{x}_1} &= 1 - \frac{1}{2} R_{A_1} \\
R_{m_1} &= 1 + \frac{N_f - 2(1+N_c)}{2N_f} R_{A_1} \\
R_{m_2} &= 5 - \frac{N_f + 2 - 2N_c(N_f-1)}{2N_f} R_{A_1} \\
R_{\widetilde{p}_2} &= -3 + \frac{1-2N_c}{2} R_{A_1} \\
R_{p_3} &= 6 + 2N_c R_{A_1}.
\end{aligned} \tag{3.4}$$

Equivalently, we could have obtained these expressions simply by considering the linear combination $R_i = R'_i - R_{A_1} X_i$, where X_i and R'_i are the $\mathrm{U}(1)_X$ and $\mathrm{U}(1)'_R$ charges of each field as given in table 3.

In order to determine R_{A_1} we should then maximize the function

$$\begin{aligned}
a^{\text{dual}}(R_{A_1}) = \frac{3}{32}\Bigg[& 2(N_f+1)(2N_f+3) + 2(N_c+2)(2N_c+5) \\
& + (N_c+2)(2N_c+5)\left(3(R_{A_1}-1)^3 - (R_{A_1}-1)\right) \\
& + (2N_f+2)(2N_f)\left(3\left(\frac{1+N_c}{N_f}R_{A_1}-1\right)^3 - \left(\frac{1+N_c}{N_f}R_{A_1}-1\right)\right) \\
& + (N_f)(2N_f-1)\left(3\left(1-2\frac{1+N_c}{N_f}R_{A_1}\right)^3 - \left(1-2\frac{1+N_c}{N_f}R_{A_1}\right)\right) \\
& + (2N_f+2)(2N_c+5)\left(3\left(-\frac{1}{2}R_{A_1}\right)^3 - \left(-\frac{1}{2}R_{A_1}\right)\right) \\
& + (2N_c+5)(2N_f)\left(3\left(\frac{N_f-2(1+N_c)}{2N_f}R_{A_1}\right)^3 - \left(\frac{N_f-2(1+N_c)}{2N_f}R_{A_1}\right)\right) \\
& + (2N_f)\left(3\left(4-\frac{N_f+2-2N_c(N_f-1)}{2N_f}R_{A_1}\right)^3 - \left(4-\frac{N_f+2-2N_c(N_f-1)}{2N_f}R_{A_1}\right)\right) \\
& + (2N_f+2)\left(3\left(-4+\frac{1-2N_c}{2}R_{A_1}\right)^3 - \left(-4+\frac{1-2N_c}{2}R_{A_1}\right)\right) \\
& + \left(3(5+2N_cR_{A_1})^3 - (5+2N_cR_{A_1})\right) \Bigg].
\end{aligned} \tag{3.5}$$

Remarkably, the solution to $da^{\text{dual}}/dR_{A_1} = 0$ corresponding to the maximum is

$$R_{A_1} = \frac{-3(1+2N_c)N_f^2 + \sqrt{16(1+N_c)^3(3+5N_c)N_f^2 - (3+4N_c(2+N_c))N_f^4}}{12(1+N_c)^3 - 3(1+2N_c)N_f^2}, \tag{3.6}$$

JHEP11(2009)049

which coincides exactly with eq. (2.5)! Moreover, the function eq. (3.5) is precisely equal to eq. (2.4). Of course, one may view this as a consequence of the fact that the 't Hooft anomalies of the dual description were designed to match those of the original theory. Nevertheless, we can perhaps view this agreement as a non-trivial check of the *dynamical* assumptions that both gauge groups are interacting and that the full superpotential eq. (3.1) is marginal, at least for the values of N_f that do not lead to violations of the unitarity bound.

Since the function $a^{\rm dual}(R_{A_1})$ is the same as before, the operators TrA_1^k hit the unitarity bound at the same thresholds as before. In particular, TrA_1^2 becomes a free field for $N_f \leq 2(N_c+1)\sqrt{\frac{1+N_c}{7+10N_c}}$. Below this threshold we can maximize

$$a_2^{\rm dual}(R_{A_1}) = a^{\rm dual}(R_{A_1}) + \frac{1}{96}(2-6R_{A_1})^2(5-6R_{A_1}). \tag{3.7}$$

This then yields the same result as eq. (2.10). The precise agreement between the dual description and the original description then continues, with TrA_1^4 becoming free at the threshold given in eq. (2.11) and TrA_1^6 becoming free at the threshold given in eq. (2.16) (for $N_c \leq 28$).

Now, one might worry that the situation changes when M_0 violates the unitarity bound and becomes a free field. This occurs either below the threshold given in eq. (2.17) or eq. (2.21), depending on the value of N_c (or at $N_f = 1$ for $N_c = 8$). When this happens, the superpotential term $M_0\widetilde{Q}\widetilde{Q}$ must be flowing to zero, since this is M_0's only interaction. In addition, for N_f just above this threshold, we know that $R_{\widetilde{Q}} \approx 2/3$ due to the superpotential interaction. A possible interpretation of this is that the $\mathrm{Sp}(2N_f+2)$ gauge group is becoming free. Under this interpretation, the coupling is flowing to zero because unitarity is now enforcing the condition that $R_{\widetilde{Q}} > 2/3$. Note that in this case one would still expect the $\mathrm{SO}(2N_c+5)$ gauge group to be strongly coupled. If this interpretation is correct, this would be similar to the mixed phase argued to exist in $\mathrm{SU}(N_c)$ gauge theories with an anti-symmetric tensor [17, 18]. In addition, there would be accidental symmetries emerging that would invalidate the a-maximization analysis performed in the electric description of the theory.

However, we will now argue that this scenario cannot be correct. To do this we will consider the sign of the $\mathrm{Sp}(2N_f+2)$ β-function in the hypothetical mixed phase. As mentioned above, since we are assuming that $\mathrm{Sp}(2N_f+2)$ is becoming free we must now have $R_{\widetilde{Q}} > 2/3$ and $R_{\widetilde{p}_2} > 2/3$ by unitarity, and thus we know that the couplings $m_2\widetilde{Q}\widetilde{p}_2$ and $(\widetilde{x}_1\widetilde{p}_2)(\widetilde{x}_1\widetilde{p}_2)p_3$ must also become irrelevant. The interacting sector of the theory then simply consists of the fields $\{\widetilde{x}_1, A_1, m_1, \widetilde{Q}\}$, with superpotential

$$W_{\rm mixed} = A_1\widetilde{x}_1\widetilde{x}_1 + m_1\widetilde{Q}\widetilde{x}_1. \tag{3.8}$$

Now, in order for this scenario to be plausible the $\mathrm{Sp}(2N_f+2)$ β-function should be positive so that the $g_{\rm Sp} \to 0$ fixed point is IR attractive. This then requires that $Tr[\mathrm{U}(1)_R\mathrm{Sp}(2N_f+2)^2] < 0$, or more explicitly

$$(2N_f+4) + (2N_f)(R_{\widetilde{Q}}-1) + (2N_c+5)(R_{\widetilde{x}_1}-1) + (2/3-1) < 0. \tag{3.9}$$

Because the superpotential and anomaly cancellation constraints lead to the same parametrization of the R-charges (for the interacting fields) as was given in eq. (3.4), this condition is equivalent to

$$R_{A_1} < -\frac{2}{3}\left(\frac{11}{2N_c-1}\right). \tag{3.10}$$

Since we expect the theory to have $R_{A_1} > 0$ so as to avoid an infinite number of free operators, this bound will never be satisfied. For example, in the limit of large N_c we obtain

$$R_{A_1} \simeq \frac{\sqrt{5}}{3}\frac{N_f}{N_c} + O(1/N_c^2), \tag{3.11}$$

and the full calculation gives qualitatively similar results. Note that it can be also verified that no subset of the couplings in eq. 3.8 leads to an IR stable fixed point. Thus, we conclude that $g_{Sp} \to 0$ is not an IR attractive fixed point in the hypothetical mixed phase, and that both gauge groups must remain interacting even after M_0 becomes free.[2]

4 Second deconfined dual description

Now we will consider the second dual description constructed in [29], which can be obtained by treating the anti-symmetric tensor A_1 as a meson of a confining $\mathrm{Sp}(2N_c{+}2)$ gauge theory, and then dualizing the $\mathrm{SO}(2N_c+5)$ gauge group using the known duality for $\mathrm{SO}(N)$ gauge theories with fundamentals [26, 37]. The field content (after integrating out massive fields) is given in table 4. In addition, the theory has the superpotential

$$\begin{aligned} W = {} & M_0(\widetilde{\widetilde{x}}_1\widetilde{m}_1)(\widetilde{\widetilde{x}}_1\widetilde{m}_1) + (\widetilde{\widetilde{x}}_1\widetilde{x}_2)(\widetilde{\widetilde{x}}_1\widetilde{x}_2) + m_2\widetilde{p}_2(\widetilde{\widetilde{x}}_1\widetilde{m}_1) + n_1\widetilde{p}_2^2 p_3 \\ & + n_1\widetilde{\widetilde{x}}_1\widetilde{\widetilde{x}}_1 + A_2\widetilde{x}_2\widetilde{x}_2 + M_1\widetilde{m}_1\widetilde{m}_1 + n_3\widetilde{r}_2\widetilde{r}_2 \\ & + n_2\widetilde{x}_2\widetilde{m}_1 + n_4\widetilde{\widetilde{x}}_1\widetilde{r}_2 + n_5\widetilde{m}_1\widetilde{r}_2, \end{aligned} \tag{4.1}$$

and the gauge-invariant operators of the electric theory match onto the operators

$$\begin{aligned} TrA^{2k} & \to TrA_2^{2k} \\ QQ & \to M_0 \\ QAQ & \to M_1 \\ QA^kQ & \to n_2A_2^{k-2}n_2, \quad k \geq 2. \end{aligned} \tag{4.2}$$

We can again begin by assuming that each of the $\mathrm{Sp}(2N_f+2)\times\mathrm{SO}(4N_f+4)\times\mathrm{Sp}(2N_c+2)$ gauge groups are interacting and that the entire superpotential in eq. 4.1 is marginal. This gives 3 constraints from anomaly cancellation and 11 constraints from the superpotential on 15 unknown R-charges, causing $a^{dual_2}(R_i)$ to again be a function of a single variable

[2] It is also interesting to note that entering the hypothetical mixed phase would have required violating the (stronger) conjecture of ref. [36] that operators with $R > 5/3$ cannot become free fields, since $R_{p_3} > 6$ in the interacting scenario.

	$\mathrm{Sp}(2N_f+2)$	$\mathrm{SO}(4N_f+4)$	$\mathrm{Sp}(2N_c+2)$	$\mathrm{SU}(2N_f)$	$\mathrm{U}(1)$	$\mathrm{U}(1)'_R$
M_0	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$2\frac{N_c+1}{N_f}$	2
$\widetilde{\widetilde{x}}_1$	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{1}{2}$	0
n_1	$\square\!\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	1	2
$\widetilde{x}_2$	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$	$\frac{1}{2}$	1
A_2	$\mathbf{1}$	$\mathbf{1}$	$\square\!\square$	$\mathbf{1}$	-1	0
$\widetilde{m}_1$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\overline{\square}$	$\frac{1}{2}-\frac{N_c+1}{N_f}$	0
M_1	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square\!\square$	$2\frac{N_c+1}{N_f}-1$	2
n_2	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\frac{N_c+1}{N_f}-1$	1
n_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$-2N_c-4$	0
n_4	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$-N_c-\frac{3}{2}$	1
n_5	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\frac{N_c+1}{N_f}-\frac{5}{2}-N_c$	1
m_2	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\frac{N_c+1}{N_f}+\frac{1}{2}-N_c$	5
$\widetilde{p}_2$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{1}{2}+N_c$	-3
p_3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$-2N_c$	6
$\widetilde{r}_2$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	N_c+2	1

Table 4. Field content of the second dual description.

(as expected). The R-charges of each field are then easily obtained in terms of R_{A_2} by considering the linear combination

$$R_i[R_{A_2}] = R'_i - X_i R_{A_2}, \tag{4.3}$$

where R'_i and X_i are the $\mathrm{U}(1)'_R$ and $\mathrm{U}(1)_X$ charges given in table 4. Alternatively, this parametrization could be obtained by using the 14 constraints to solve for the 15 unknown R-charges in terms R_{A_2}, as we did in the previous section.

It is then straightforward to verify that the function

$$a^{dual_2}(R_{A_2}) = \frac{3}{32}\Bigg[2(N_f+1)(2N_f+3)+2(2N_f+2)(4N_f+3)+2(N_c+1)(2N_c+3) + \sum_i \dim_{\mathcal{O}_i}\big(3(R_i[R_{A_2}]-1)^3-(R_i[R_{A_2}]-1)\big)\Bigg] \tag{4.4}$$

is exactly equal to the functions given in eqs. (2.4) and (3.5). In particular, maximizing it gives rise to the same $\mathrm{U}(1)_R$ symmetry as was found in the previous sections. Furthermore, the operators TrA_2^k become free fields at the same thresholds as before as we lower N_f from $2(N_c+1)$.[3]

[3] It is perhaps worrisome that the gauge-invariant operator $n_4\widetilde{p}_2$ appears to badly violate the unitarity bound, since it has $R_{n_4\widetilde{p}_2} = -2+2R_{A_2}$. However, if the duality is to be believed, non-perturbative effects in this description should cause this operator to be zero in the chiral ring in order to avoid a contradiction.

JHEP11(2009)049

When the operator M_0 hits the unitarity bound we assume that the coupling $M_0(\widetilde{\widetilde{x}}_1\widetilde{m}_1)(\widetilde{\widetilde{x}}_1\widetilde{m}_1)$ is simply flowing to zero so that M_0 can become a free field. Note that this is unlikely be the result of the SO$(4N_f+4)$ gauge coupling flowing to zero because this would also force M_1 to be a free field and one would expect that $R_{M_1} \approx 2/3$ close to this threshold, which is not the case. Furthermore, the Sp$(2N_c+2)$ gauge group going free would not cause this operator to become irrelevant.

Thus we first consider the possibility that, similar to the hypothetical mixed phase considered in the previous section, the Sp$(2N_f+2)$ gauge group is becoming free when M_0 hits the unitarity bound. If this is the case, unitarity requires that $R_{n_1}, R_{n_4}, R_{\widetilde{p}_2} > 2/3$ and forces the couplings $m_2\widetilde{p}_2(\widetilde{\widetilde{x}}_1\widetilde{m}_1)$ and $n_1\widetilde{p}_2^2 p_3$ to become irrelevant. The interacting superpotential of the mixed phase then becomes

$$W_{\text{mixed}} = (\widetilde{\widetilde{x}}_1\widetilde{x}_2)(\widetilde{\widetilde{x}}_1\widetilde{x}_2) + n_1\widetilde{\widetilde{x}}_1\widetilde{\widetilde{x}}_1 + A_2\widetilde{x}_2\widetilde{x}_2 + M_1\widetilde{m}_1\widetilde{m}_1 \\ + n_3\widetilde{r}_2\widetilde{r}_2 + n_2\widetilde{x}_2\widetilde{m}_1 + n_4\widetilde{\widetilde{x}}_1\widetilde{r}_2 + n_5\widetilde{m}_1\widetilde{r}_2, \tag{4.5}$$

along with the free fields $\{M_0, m_2, \widetilde{p}_2, p_3\}$ and potentially free operators TrA_2^{2k}.

The superpotential combined with anomaly cancellation then give 10 constraints on 11 unknown R-charges, with the same parametrization for the R-charges of the interacting fields as in eq. 4.3. However, again we can rewrite the condition that $Tr[\text{U}(1)_R\text{Sp}(2N_f+2)^2] < 0$ as

$$R_{A_2} < -\frac{2}{3}\left(\frac{11}{2N_c-1}\right), \tag{4.6}$$

and this scenario is disfavored for the same reason as in first dual. It is straightforward to additionally verify that no subset of the couplings in eq. 4.5 lead to an IR attractive fixed point.

Next we would like to investigate the possibility that when M_1 becomes a free field the SO$(4N_f+4)$ gauge coupling is flowing to zero. Note that in this case unitarity is forcing the $M_1\widetilde{m}_1\widetilde{m}_1$ coupling to become irrelevant. To see if this is plausible we can again attempt to determine the U$(1)_R$ symmetry of the hypothetical mixed phase and check the sign of the SO$(4N_f+4)$ β-function.

If it is correct that the SO$(4N_f+4)$ gauge coupling flows to zero, unitarity also requires that $R_{\widetilde{r}_2} > 2/3$ in addition to $R_{\widetilde{m}_1} > 2/3$. These conditions imply that the couplings $m_2\widetilde{p}_2(\widetilde{\widetilde{x}}_1\widetilde{m}_1)$, $n_3\widetilde{r}_2\widetilde{r}_2$, and $n_5\widetilde{m}_1\widetilde{r}_2$ should become irrelevant, and thus it is reasonable to assume that the interacting superpotential becomes

$$W_{\text{mixed}} = (\widetilde{\widetilde{x}}_1\widetilde{x}_2)(\widetilde{\widetilde{x}}_1\widetilde{x}_2) + n_1\widetilde{p}_2^2 p_3 + n_1\widetilde{\widetilde{x}}_1\widetilde{\widetilde{x}}_1 + A_2\widetilde{x}_2\widetilde{x}_2 + n_2\widetilde{x}_2\widetilde{m}_1 + n_4\widetilde{\widetilde{x}}_1\widetilde{r}_2, \tag{4.7}$$

along with the free fields $\{M_0, M_1, n_3, n_5, m_2\}$ and potentially free operators TrA_2^{2k}. Since we now have 8 constraints and 10 unknown R-charges, $a(R_i)$ will be a function of 2 variables and is best maximized numerically. Doing this, however, we find that the function has no stable maximum and thus the fixed point does not exist.

If we turn off one additional coupling, we find that the only IR stable choice is to assume that $(\widetilde{\widetilde{x}}_1\widetilde{x}_2)(\widetilde{\widetilde{x}}_1\widetilde{x}_2)$ is flowing to zero — i.e., only this operator has $R > 2$ in the

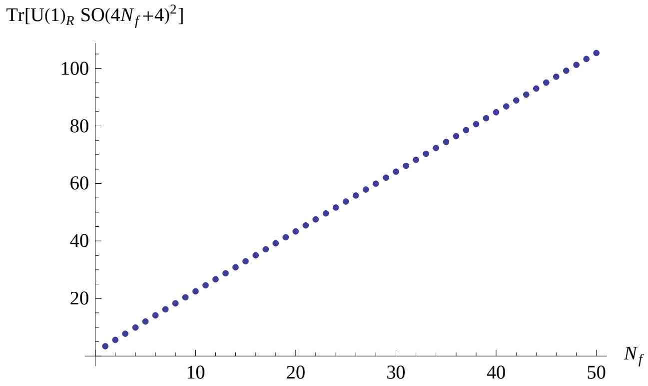

Figure 2. $Tr[\mathrm{U}(1)_R \mathrm{SO}(4N_f + 4)^2]$ as a function of N_f in the large N_c limit of the hypothetical mixed phase. Because it is always positive the $\mathrm{SO}(4N_f + 4)$ gauge coupling can not flow to zero and the mixed phase does not exist.

hypothetical CFT in which it is absent from the superpotential. However, in this case one can then check that $Tr[\mathrm{U}(1)_R \mathrm{SO}(4N_f+4)] > 0$ for all N_c and N_f, and hence the assumption that $g_{SO} \to 0$ is not correct. To illustrate this, in figure 2 we plot $Tr[\mathrm{U}(1)_R \mathrm{SO}(4N_f + 4)^2]$ as a function of N_f in the limit of large N_c. We have also checked that no subset of these couplings leads to an IR stable fixed point. We thus do not find any evidence for a mixed phase in this description of the theory.

5 Conclusions

In this work we have attempted to map out the phase structure of supersymmetric $\mathrm{Sp}(2N_c)$ gauge theories with $2N_f$ fundamentals, an adjoint, and vanishing superpotential. The IR behavior of this theory has an incredibly rich structure and has previously been difficult to analyze. Using a-maximization, however, we have been able to check the conjectures of [29] as well as look for evidence that the theory enters a mixed phase below some value of N_f. We have not found any such evidence in the simplest known dual descriptions of the theory. It is thus tempting to believe (though far from proven) that the original electric description of the theory is a good description for all N_c and N_f.

A straightforward extension of the present work would be to construct the deconfined dual descriptions of the $\mathrm{SU}(N_c)$ version of this theory and perform a similar analysis. It would also be quite interesting to find dual descriptions of these theories in which the operators TrA^{2k} appear as elementary fields so that one could better understand the way in which they decouple from the theory. More generally, it would be interesting to find new examples of theories that possess mixed phases (as in [18]) so that one could better understand and classify the situations under which they can occur. These possible directions are left to future work.

Acknowledgments

We thank Jonathan Heckman, Ken Intriligator, John Mason, David Morrissey, and David Simmons-Duffin for helpful discussions and inspiration. This work is supported in part by the Harvard Center for the Fundamental Laws of Nature and by NSF grant PHY-0556111.

References

[1] E. Witten, *Dynamical breaking of supersymmetry*, *Nucl. Phys.* **B 188** (1981) 513 [SPIRES].

[2] I. Affleck, M. Dine and N. Seiberg, *Supersymmetry breaking by instantons*, *Phys. Rev. Lett.* **51** (1983) 1026 [SPIRES]; *Dynamical supersymmetry breaking in supersymmetric QCD*, *Nucl. Phys.* **B 241** (1984) 493 [SPIRES]; *Dynamical supersymmetry breaking in four-dimensions and its phenomenological implications*, *Nucl. Phys.* **B 256** (1985) 557 [SPIRES].

[3] For reviews, see:
E. Poppitz and S.P. Trivedi, *Dynamical supersymmetry breaking*, *Ann. Rev. Nucl. Part. Sci.* **48** (1998) 307 [hep-th/9803107] [SPIRES];
Y. Shadmi and Y. Shirman, *Dynamical supersymmetry breaking*, *Rev. Mod. Phys.* **72** (2000) 25 [hep-th/9907225] [SPIRES];
K.A. Intriligator and N. Seiberg, *Lectures on supersymmetry breaking*, *Class. Quant. Grav.* **24** (2007) S741 [hep-ph/0702069] [SPIRES].

[4] M.A. Luty and R. Sundrum, *Supersymmetry breaking and composite extra dimensions*, *Phys. Rev.* **D 65** (2002) 066004 [hep-th/0105137] [SPIRES]; *Anomaly mediated supersymmetry breaking in four dimensions, naturally*, *Phys. Rev.* **D 67** (2003) 045007 [hep-th/0111231] [SPIRES].

[5] M. Dine et al., *Visible effects of the hidden sector*, *Phys. Rev.* **D 70** (2004) 045023 [hep-ph/0405159] [SPIRES];
R. Sundrum, *'Gaugomaly' mediated SUSY breaking and conformal sequestering*, *Phys. Rev.* **D 71** (2005) 085003 [hep-th/0406012] [SPIRES];
M. Ibe, K.I. Izawa, Y. Nakayama, Y. Shinbara and T. Yanagida, *Conformally sequestered SUSY breaking in vector-like gauge theories*, *Phys. Rev.* **D 73** (2006) 015004 [hep-ph/0506023] [SPIRES]; *More on conformally sequestered SUSY breaking*, *Phys. Rev.* **D 73** (2006) 035012 [hep-ph/0509229] [SPIRES];
M. Schmaltz and R. Sundrum, *Conformal sequestering simplified*, *JHEP* **11** (2006) 011 [hep-th/0608051] [SPIRES];
S. Kachru, L. McAllister and R. Sundrum, *Sequestering in string theory*, *JHEP* **10** (2007) 013 [hep-th/0703105] [SPIRES].

[6] T.S. Roy and M. Schmaltz, *A hidden solution to the μ/B_μ problem in gauge mediation*, *Phys. Rev.* **D 77** (2008) 095008 [arXiv:0708.3593] [SPIRES].

[7] H. Murayama, Y. Nomura and D. Poland, *More visible effects of the hidden sector*, *Phys. Rev.* **D 77** (2008) 015005 [arXiv:0709.0775] [SPIRES].

[8] G. Perez, T.S. Roy and M. Schmaltz, *Phenomenology of SUSY with scalar sequestering*, *Phys. Rev.* **D 79** (2009) 095016 [arXiv:0811.3206] [SPIRES];
H.D. Kim and J.-H. Kim, *Higgs phenomenology of scalar sequestering*, arXiv:0903.0025 [SPIRES];

N.J. Craig and D. Green, *On the phenomenology of strongly coupled hidden sectors*, *JHEP* **09** (2009) 113 [arXiv:0905.4088] [SPIRES].

[9] A.E. Nelson and M.J. Strassler, *Suppressing flavor anarchy*, *JHEP* **09** (2000) 030 [hep-ph/0006251] [SPIRES]; *Exact results for supersymmetric renormalization and the supersymmetric flavor problem*, *JHEP* **07** (2002) 021 [hep-ph/0104051] [SPIRES].

[10] T. Kobayashi, H. Nakano and H. Terao, *Yukawa hierarchy transfer from superconformal sector and degenerate sfermion masses*, *Phys. Rev.* **D 65** (2002) 015006 [hep-ph/0107030] [SPIRES];
T. Kobayashi, H. Nakano, T. Noguchi and H. Terao, *Sfermion mass degeneracy, superconformal dynamics and supersymmetric grand unified theories*, *Phys. Rev.* **D 66** (2002) 095011 [hep-ph/0202023] [SPIRES].

[11] D. Poland and D. Simmons-Duffin, *Superconformal flavor simplified*, arXiv:0910.4585 [SPIRES].

[12] T. Yanagida, *Naturally light Higgs doublets in the supersymmetric grand unified theories with dynamical symmetry breaking*, *Phys. Lett.* **B 344** (1995) 211 [hep-ph/9409329] [SPIRES];
T. Hotta, K.I. Izawa and T. Yanagida, *Dynamical models for light higgs doublets in supersymmetric grand unified theories*, *Phys. Rev.* **D 53** (1996) 3913 [hep-ph/9509201] [SPIRES];
J. Hisano and T. Yanagida, *An $N = 2$ SUSY gauge model for dynamical breaking of the grand unified* SU(5) *symmetry*, *Mod. Phys. Lett.* **A 10** (1995) 3097 [hep-ph/9510277] [SPIRES];
T. Hotta, K.I. Izawa and T. Yanagida, *Quantum restoration of the* $\mathrm{U}(1)_Y$ *symmetry in dynamically broken SUSY-GUT's*, *Phys. Lett.* **B 409** (1997) 245 [hep-ph/9511431] [SPIRES]; *Nonabelian duality and higgs multiplets in supersymmetric grand unified theories*, *Prog. Theor. Phys.* **95** (1996) 949 [hep-ph/9601320] [SPIRES].

[13] T. Hotta, K.I. Izawa and T. Yanagida, *Natural unification with a supersymmetric* $\mathrm{SO}(10)_{\mathrm{GUT}} \times \mathrm{SO}(6)_H$ *gauge theory*, *Phys. Rev.* **D 54** (1996) 6970 [hep-ph/9602439] [SPIRES];
K.I. Izawa and T. Yanagida, *R-invariant natural unification*, *Prog. Theor. Phys.* **97** (1997) 913 [hep-ph/9703350] [SPIRES]; *R-invariant unification with dynamical higgs multiplets*, *Prog. Theor. Phys.* **99** (1998) 423 [hep-ph/9710218] [SPIRES].

[14] H.-C. Cheng, *Supersymmetric dynamical generation of the grand unification scale*, *Phys. Lett.* **B 410** (1997) 45 [hep-ph/9702214] [SPIRES];
M. Graesser, *Getting the supersymmetric unification scale from quantum confinement with chiral symmetry breaking*, *Phys. Rev.* **D 59** (1999) 035007 [hep-ph/9805417] [SPIRES];
R. Kitano and G.D. Kribs, *Tripletless unification in the conformal window*, *JHEP* **03** (2005) 033 [hep-ph/0501047] [SPIRES];
Y. Nomura, D. Poland and B. Tweedie, *Holographic grand unification*, *JHEP* **12** (2006) 002 [hep-ph/0605014] [SPIRES];
R. Kitano, *Dynamical GUT breaking and mu-term driven supersymmetry breaking*, *Phys. Rev.* **D 74** (2006) 115002 [hep-ph/0606129] [SPIRES].

[15] K.A. Intriligator and N. Seiberg, *Lectures on supersymmetric gauge theories and electric-magnetic duality*, *Nucl. Phys. Proc. Suppl.* **45BC** (1996) 1 [hep-th/9509066] [SPIRES].

[16] J. Terning, *Non-perturbative supersymmetry*, hep-th/0306119 [SPIRES].

[17] J. Terning, *Duals for* SU(N) *SUSY gauge theories with an antisymmetric tensor: five easy flavors*, *Phys. Lett.* **B 422** (1998) 149 [hep-th/9712167] [SPIRES].

[18] C. Csáki, P. Meade and J. Terning, *A mixed phase of SUSY gauge theories from a-maximization*, *JHEP* **04** (2004) 040 [hep-th/0403062] [SPIRES].

[19] K.A. Intriligator and B. Wecht, *The exact superconformal R-symmetry maximizes a*, *Nucl. Phys.* **B 667** (2003) 183 [hep-th/0304128] [SPIRES].

[20] D. Kutasov, A. Parnachev and D.A. Sahakyan, *Central charges and* $U(1)_R$ *symmetries in* $N = 1$ *super Yang- Mills*, *JHEP* **11** (2003) 013 [hep-th/0308071] [SPIRES].

[21] D. Kutasov, *New results on the 'a-theorem' in four dimensional supersymmetric field theory*, hep-th/0312098 [SPIRES].

[22] E. Barnes, K.A. Intriligator, B. Wecht and J. Wright, *Evidence for the strongest version of the 4D a-theorem, via a-maximization along RG flows*, *Nucl. Phys.* **B 702** (2004) 131 [hep-th/0408156] [SPIRES].

[23] R.G. Leigh and M.J. Strassler, *Accidental symmetries and* $N = 1$ *duality in supersymmetric gauge theory*, *Nucl. Phys.* **B 496** (1997) 132 [hep-th/9611020] [SPIRES].

[24] G. Mack, *All unitary ray representations of the conformal group* $SU(2, 2)$ *with positive energy*, *Commun. Math. Phys.* **55** (1977) 1 [SPIRES];
M. Flato and C. Fronsdal, *Representations of conformal supersymmetry*, *Lett. Math. Phys.* **8** (1984) 159 [SPIRES];
V.K. Dobrev and V.B. Petkova, *All positive energy unitary irreducible representations of extended conformal supersymmetry*, *Phys. Lett.* **B 162** (1985) 127 [SPIRES].

[25] D. Anselmi, J. Erlich, D.Z. Freedman and A.A. Johansen, *Positivity constraints on anomalies in supersymmetric gauge theories*, *Phys. Rev.* **D 57** (1998) 7570 [hep-th/9711035] [SPIRES].

[26] N. Seiberg, *Electric-magnetic duality in supersymmetric nonAbelian gauge theories*, *Nucl. Phys.* **B 435** (1995) 129 [hep-th/9411149] [SPIRES].

[27] R.G. Leigh and M.J. Strassler, *Duality of* $Sp(2N_c)$ *and* $SO(N_c)$ *supersymmetric gauge theories with adjoint matter*, *Phys. Lett.* **B 356** (1995) 492 [hep-th/9505088] [SPIRES].

[28] M. Berkooz, *The dual of supersymmetric SU(2k) with an antisymmetric tensor and composite dualities*, *Nucl. Phys.* **B 452** (1995) 513 [hep-th/9505067] [SPIRES].

[29] M.A. Luty, M. Schmaltz and J. Terning, *A sequence of duals for* $Sp(2N)$ *supersymmetric gauge theories with adjoint matter*, *Phys. Rev.* **D 54** (1996) 7815 [hep-th/9603034] [SPIRES].

[30] K.A. Intriligator and B. Wecht, *RG fixed points and flows in SQCD with adjoints*, *Nucl. Phys.* **B 677** (2004) 223 [hep-th/0309201] [SPIRES].

[31] E. Barnes, K.A. Intriligator, B. Wecht and J. Wright, $N = 1$ *RG flows, product groups and a-maximization*, *Nucl. Phys.* **B 716** (2005) 33 [hep-th/0502049] [SPIRES].

[32] T. Kawano, Y. Ookouchi, Y. Tachikawa and F. Yagi, *Pouliot type duality via a-maximization*, *Nucl. Phys.* **B 735** (2006) 1 [hep-th/0509230] [SPIRES].

[33] T. Kawano and F. Yagi, *Supersymmetric* $N = 1$ *Spin*(10) *gauge theory with two spinors via a-maximization*, *Nucl. Phys.* **B 786** (2007) 135 [arXiv:0705.4022] [SPIRES].

[34] K.A. Intriligator and P. Pouliot, *Exact superpotentials, quantum vacua and duality in supersymmetric* $SP(N_c)$ *gauge theories*, *Phys. Lett.* **B 353** (1995) 471 [hep-th/9505006] [SPIRES].

[35] E. Poppitz, Y. Shadmi and S.P. Trivedi, *Duality and exact results in product group theories*, *Nucl. Phys.* **B 480** (1996) 125 [hep-th/9605113] [SPIRES].

[36] K.A. Intriligator, *IR free or interacting? A proposed diagnostic*, *Nucl. Phys.* **B 730** (2005) 239 [hep-th/0509085] [SPIRES].

[37] K.A. Intriligator and N. Seiberg, *Duality, monopoles, dyons, confinement and oblique confinement in supersymmetric* $\mathrm{SO}(N_c)$ *gauge theories*, *Nucl. Phys.* **B 444** (1995) 125 [hep-th/9503179] [SPIRES].